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AN UPPER BOUND ON RANDOM BUFFETING FORCES CAUSED BY TWO-PHASE FLOWS ACROSS TUBES

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This paper investigates the random buffeting excitation forces that apply to tubes in two-phase cross-flows. The spectral magnitude of these forces is studied with the help of a database that includes most of the available experimental data on direct or indirect force measurements. Different fluid mixtures, namely steam–water, air–water and various types of Freon, as well as different thermohydraulic or geometrical situations are included in the database. Using a formalism similar in principle to that used successfully in the modelling of buffeting in single-phase flows, the scaling of the data is undertaken. While dynamic pressure, viscosity or surface tension are found not to be relevant parameters, gravity forces allow us to define appropriate dimensionless spectra for all cases. The meaning of these parameters as well as the effects of flow regimes or fluid mixtures are discussed. Finally, an upper bound on the magnitude of these forces, which is of practical applicability, is proposed.

1. INTRODUCTION

Many engineering systems operate with two-phase flows across tube bundles for heat exchange purposes. The tendency to increase efficiency through higher mass fluxes or thinner tubes may lead to vibrations and damage. Such flow-induced vibrations are known to originate via several distinct mechanisms, such as turbulence, vortex shedding or fluid-elastic forces. In recent years, most of the attention has been focused on fluid-elastic forces which may be the cause of short-term failures through instabilities and high amplitude vibrations, when the supporting or damping of the tubes is inadequate. These fluid-elastic forces are usually defined as those that depend on the motion of the vibrating body. Yet, some forces exist, of a lesser magnitude, that are entirely governed by the fluctuations inherent to the flow. They may cause small random vibrations of the tubes and are often referred to as random buffeting forces. These vibrations due to turbulence-like forces need to be taken into account in predictive analysis, because they might induce progressive long-term damage at the tube supports through wear or fatigue. A typical example is that of a U-tube in a steam generator undergoing out-of-plane motion as a consequence of fluid-elastic instability. Whereas the level of resulting impacts on the antivibration devices are governed by the

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fluid-elastic forces, potential wear will also depend on the random in-plane motion caused by buffeting forces.

In single-phase flows, these random forces have been extensively measured and analysed. They have been related to the turbulence level created by the bundle itself. Experimental data obtained for different kinds of fluids and tube bundles have been satisfactorily compared through the use of adequate data-reduction procedures. This has naturally led to the definition of upper bounds for these forces, which are now extensively used in industrial applications (Axisa *et al.* 1990; Blevins 1991).

In the case of two-phase flows, such an extensive analysis has not yet been undertaken, though it is known that there are significant differences between single- and two-phase situations. Large sets of experimental results have been published in the past 20 years, with a noticeable increase in recent years. Reduction of parts of these data have been proposed (Axisa *et al.* 1990; Axisa & Villard 1992; Nakamura *et al.* 1992; 1995; Taylor 1992; Taylor *et al.* 1996; de Langre & Villard 1994; de Langre *et al.* 1995). Though significant progress has been made in these analyses, many questions remain open such as the effects of viscosity, surface tension, density ratio or flow regimes (e.g. bubbly or intermittent). Actually, the main unsolved problem is that of the physical mechanism that induces these forces.

Yet, leaving the last question unanswered, there is a practical need for the designer of heat exchangers to be aware of the magnitude of random forces caused by two-phase flows. In this paper, following an approach similar to that of Axisa *et al.* (1990), we propose to define an upper bound for the spectrum of these forces. This upper bound needs to be known with a sufficient level of confidence that it can be used in predictive analysis. For that purpose, a large set of experimental published data will be considered here.

2. THEORETICAL BACKGROUND

2.1. Description of the Two-Phase Flow Model

Liquid and gaseous phases have indeed complex interactions, especially when flowing across tube arrays. These interactions are the subject of intensive research in heat transfer studies or for the prediction of flow-induced vibrations. For the purpose of this paper, we only consider at first the simplest and most commonly used model to describe the flowing mixture. A homogeneous flow model is used, whereby the two phases have a common velocity. The associated homogeneous void fraction is defined as

$$\alpha = \frac{Q_g}{Q_g + Q_l}, \tag{1}$$

where Q stands for the volume flow rate, and subscripts g and l identify the gas or liquid phase. The mixture density ρ is derived from the densities of the two phases via the formula

$$\rho = \alpha\rho_g + (1 - \alpha)\rho_l, \tag{2}$$

The gap flow velocity V is scaled with respect to the homogeneous free-stream cross-flow velocity as

$$V = \frac{P}{P - D}\frac{\rho_g Q_g + \rho_l Q_l}{\rho A}, \tag{3}$$

where A is the free-stream cross-sectional area, D is the diameter of the tube and P is the pitch of the array. Differences that may arise in terms of two-phase flow regimes are not taken into account here, but are discussed further later.

2.2. Spectrum of the Fluctuating Forces

Following the analysis of Axisa *et al.* (1990), it is assumed that fluctuating forces at the tube wall induce a random force per unit length $F(s, t)$, which is entirely characterized by its cross-correlation spectrum,

$$\Psi_F(s_1, s_2, f) = \int_{-\infty}^{+\infty} \left(\lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{+T} F(s_1, t) F(s_2, t + \tau)\, \mathrm{d}t \right) \mathrm{e}^{-2\mathrm{i}\pi f \tau}\, \mathrm{d}\tau. \tag{4}$$

Rather than modelling F, simplifying assumptions are made on Ψ_F by separating the dependence on space variables (s_1, s_2) and frequency f and by characterizing the space dependence through a correlation length λ_c, such that

$$\Psi_F(s_1, s_2, f) = \Phi(f)\mathrm{e}^{-|s_2 - s_1|/\lambda_c}, \tag{5}$$

where Φ is the autocorrelation spectrum of forces per unit length. The vibratory response of a tube to such a force is easily obtained using classical random vibration theory, if it is assumed that the fluid excitation is random, ergodic and sufficiently broad-banded in the vicinity of the tube natural frequency. In the case of small structural damping, and considering here a uniform cross-flow for the sake of clarity, the r.m.s. displacement contributed by a given vibration mode n is

$$y_n(s) = \left[\frac{\varphi_n^2(s) L \lambda_c a_n}{64\pi^3 f_n^3 M_n^2 \zeta_n} \Phi(f_n) \right]^{1/2}, \tag{6}$$

where L, φ_n, M_n, f_n and ζ_n are the tube length, modal shape, mass, frequency, and damping, respectively, and a_n is the modal correlation factor, proportional to the joint acceptance, defined as

$$a_n = \frac{1}{\lambda_c L} \int_0^L \int_0^L [\varphi_n(s_1)\varphi_n(s_2)\mathrm{e}^{-|s_2 - s_1|/\lambda_c}]\, \mathrm{d}s_1\, \mathrm{d}s_2. \tag{7}$$

If λ_c/L is small enough (de Langre *et al.* 1991), this coefficient may be approximated as

$$a_n = \frac{2}{L} \int_0^L \varphi_n^2(s)\, \mathrm{d}s, \tag{8}$$

so that in equation (6) the dependence on λ_c may be separated:

$$y_n(s) = \left[\frac{\varphi_n^2(s) L^2 a_n}{64\pi^3 f_n^3 M_n^2 \zeta_n} \left(\frac{\lambda_c}{L} \Phi(f_n) \right) \right]^{1/2}. \tag{9}$$

In the above equation, it is seen that the only spectrum which needs to be known for the calculation of the vibratory response is the equivalent spectrum introduced by Axisa *et al.* (1990), namely

$$\Phi_E(f) = \frac{\lambda_c}{L} \Phi(f), \tag{10}$$

thus avoiding any further hypothesis on the correlation length. This equivalent spectrum can be understood as that of fully correlated random forces per unit length that would have the same effect on the tube as the forces defined in equation (4). Since the actual level of total excitation does not increase with length as it would with fully correlated forces, this equivalent spectrum is associated by nature with a reference length L_0. In a similar way a reference diameter D_0 is needed to take into account the growth of forces with the surface of the tube. The relation between equivalent spectra in different tube geometries is obtained through

$$\Phi_E(f) = \left(\frac{L_0}{L}\right)\left(\frac{D}{D_0}\right)\Phi_E^0(f), \tag{11}$$

where $\Phi_E^0(f)$ is a reference equivalent spectrum associated with arbitrary reference lengths L_0 and D_0. In this paper, we use $L_0 = 1$ m and $D_0 = 0{\cdot}20$ m as in de Langre *et al.* (1991) and Axisa & Villard (1992).

2.3. Experimental Procedure

From a practical point of view, the foregoing equations may be used to derive experimental values of the reference equivalent spectrum, starting from the buffeting response of a tube. Several experimental procedures may be used to obtain values of Φ_E^0.

The most common is to let a flexible tube vibrate under a two-phase cross-flow. Most of the parameters in equation (9) are then readily obtained through spectral analysis of the in-flow motion, provided that the modal behaviour is simple enough. If strong coupling is found between the modes of adjacent tubes, which is typically the case of flexible arrays at low void fractions, specific procedures may be needed (Granger 1990; Hadj-Sadok *et al.* 1995). One value of Φ_E^0 per mode is thus obtained, at the modal frequency $f = f_n$,

$$\Phi_E^0(f_n) = \left(\frac{L}{L_0}\right)\left(\frac{D_0}{D}\right)\frac{64\pi^3 f_n^3 M_n^2 \zeta_n}{\varphi_n^2(s) L^2 a_n}\, y_n^2. \tag{12}$$

Another method consists in a direct measurement by the use of load cells integrating the flow-induced forces on a length L of the tube (Axisa *et al.* 1990; Nakamura *et al.* 1992). In that case, the autocorrelation spectrum of the measured force is

$$\Phi_F(f) = \int_0^L \int_0^L \Psi_F(s_1, s_2, f)\, \mathrm{d}s_1\, \mathrm{d}s_2, \tag{13}$$

so that the equivalent spectrum is readily obtained using equations (4) and (11) as

$$\Phi_E^0(f) = \left(\frac{L}{L_0}\right)\left(\frac{D_0}{D}\right)\frac{1}{aL^2}\,\Phi_F(f), \tag{14}$$

where a is a correlation factor defined by equation (7) with $\varphi_n = 1$. For each flow condition, the latter procedure provides values of Φ_E^0 for the whole range of frequencies which is obtained from the load cell spectra.

2.4. Scaling Procedure

The purpose of trying to define here a dimensionless form of the reference equivalent spectrum is to isolate the governing physical parameters and thereby to relate results from one configuration to another. Two scaling parameters are needed: one for the time scale, thus defining a reduced frequency, $f_R = f/f_0$; and one for the pressure scale, p_0, thus defining a dimensionless reference equivalent spectrum,

$$\overline{\Phi_E^0}(f/f_0) = \frac{f_0}{(p_0 D)^2}\,\Phi_E^0(f). \tag{15}$$

In the case of single-phase flow, it has been shown by several authors that $f_{0} = V/D$ and $p_0 = \frac{1}{2}\rho V^2$ lead to comparable dimensionless spectra for a large variety of experimental configurations (Blevins 1991). Defining upper bounds in terms of dimensionless spectra is therefore legitimate, as their use in predictive analysis leads to an acceptable approximation of the magnitude of buffeting forces.

In two-phase flows the approach is in principle identical, but the above scaling factors have to be reconsidered (Axisa *et al.* 1990; Nakamura *et al.* 1992; Taylor 1992). The search for adequate scaling parameters may be guided by the knowledge of the physical mechanism involved in random excitation processes. This is not easily feasible today. In the work presented here, it was therefore decided to undertake a systematic search over a large set of potential scaling factors obtained from the physical parameters of the experiments. The adequate frequency and pressure scaling factors, in that sense, will be those which allow to define dimensionless spectra $\overline{\Phi_E^0}(f)$ that are comparable for all experiments. Such an approach requires the largest possible set of data to be used, as only statistical information on scatter is considered. Once a proper scaling is achieved, an upper bound can be defined which is the aim of the present work.

3. EXPERIMENTAL DATABASE

Experimental data published on this subject may be found for different kinds of fluid mixtures. In industrial applications the most common is steam-water because of the efficiency of thermal exchange near nucleate boiling. Unfortunately, the practical undertaking of vibrational experiments in the corresponding thermo-hydraulic environment requires considerable effort. Substitution mixtures have therefore been extensively used for research purpose and preliminary analyses. Air and water at room temperature and pressure has been favoured by many authors because of its suitability for low cost experiment. The use of such a mixture has often been questioned because of the strong differences with steam-water in terms of physical properties (density ratios, surface tensions, viscosities, etc.). Still, a lot of the present understanding of two-phase flow effects on vibrations has been obtained from air-water tests. More refined mixtures made up of several types of gaseous and liquid Freon, or even water and gaseous Freon, allow a better modelling of some of the aforementioned physical properties. In fact, as it is not known which parameters control buffeting forces in two-phase flow, it is not clear which is the most appropriate substitution mixture. These Freon mixtures being more difficult to use than air-water, fewer published data are available, most of them having appeared only very recently. It should be emphasized that, in the past 10 years, large sets of data using all types of mixtures have been published, which were of limited availability before.

As emphasized above, the approach used in this paper requires the analysis of the largest possible set of data. Hence, as stated in similar approaches of data analysis (Pettigrew 1991), one has to consider the large uncertainties related to incomplete published information, inconsistent definitions, or data obtained from graphs. For the scope of this work, which is to have a first-order approximation of these forces for practical applications, the number of tests and the variety of data is a priority, not the similarity and comparative accuracy of the results. The choice of the experimental data in the literature was made according to the following criteria: accessibility to the parameters used in Section 2, variety of the physical parameters and mechanical systems, variety of experimental procedures, and no over-representation of a particular fluid mixture, in particular air-water for which there is an abundance of tests.This led to the choice of 11 sets of published data.

Steam-water data are taken from research programs on three different tube bundles: (i) the EVA program (Axisa *et al.* 1984, 1985) where a straight flexible tube bundles was subjected to a steam–water mixture at 25 bar and 210°C; (ii) tests at 5, 30 and 58 bar (Nakamura *et al.* 1992, 1995) on a rigid bundle, where the resulting buffeting force is measured through a load cell; (iii) more recent tests on a flexible bundle (Hirota *et al.* 1996) at the same pressure levels.

Air-water results are widely published. Four different sets were considered, from: (i) tests with a tube mounted on a flexible plate, within a rigid bundle (Axisa *et al.* 1989); (ii) tests named CRL in Taylor (1994) and Taylor *et al.* (1996); (iii) direct measurements from a load cell (Nakamura *et al.* 1992), following a principle similar to those in steam-water in the same study.

Most of the available data involving Freon mixtures were considered: (i) tests with R-11 on a single flexible tube in a rigid array (Feenstra *et al.* 1996); (ii) tests with R-22 (Pettigrew & Taylor 1994b); (iii) tests with R-114 from the CLOTAIRE program (Axisa & Villard 1992) on U-tubes, where the response of two different modes is considered; (iv) tests with gaseous R-13B1 and water (Gay *et al.* 1988) on a tube partially exposed to the flow; and (v) tests with the same mixture on a different bundle (Delenne *et al.* 1997).

It should be noted that the measurement of buffeting forces was not the purpose of most of these tests; they were usually oriented towards the description of fluid-elastic instability. For the sake of clarity, only a few data points of each experimental program will be used in the present analysis. They were chosen at the extreme possible values of void fractions and velocities in each program, excluding situations close to fluid-elastic instability because of the corresponding uncertainties on damping. Values of lift and drag forces are not differentiated hereafter, as they are known to be of similar orders of magnitude. Table 1 summarizes the values of the parameters associated with each data point, as used in the analysis of the next section. They were either taken from the work referenced above, as communicated by the authors of the experiments, or, if not explicitly available, calculated from published data. One should consider that a design guideline approach does not require very accurate values of each parameter.

4. SPECTRAL CONTENT OF THE BUFFETING FORCES

Values of the reference equivalent spectrum $\Phi_E^0(f)$ resulting from the methodology described in the foregoing are plotted in Figure 1. It can be seen that for a given frequency the corresponding spectrum level varies in a range of about four decades when the experimental database is explored. As these experiments were done under quite different conditions in terms of mechanical systems or flow conditions, significant differences could be expected.

The above data points are now rescaled using the standard factors for single-phase flow, i.e. $f_0 = V/D$ and $p_0 = \frac{1}{2}\rho V^2$. Figure 2 clearly confirms previous observations with smaller data sets (Axisa & Villard 1992; Taylor *et al.* 1988): such factors are obviously not adequate. A large part of our data set falls above the upper bound proposed by Axisa *et al.* (1990) for single-phase flow, and in fact above all upper bounds reviewed by Blevins (1991). Use of any one of the proposed single-phase upper bound for two-phase applications would, for some range of parameters, clearly underestimate the buffeting forces, and thus the damage induced by vibrations.

The same data are now reduced using a different pressure scaling factor, which is designed to include the effect of viscosity. Though the equivalent viscosity of a two-phase mixture is not a very well defined concept, we resort here to the most commonly used approximation of McAdam (Pettigrew & Taylor 1994a), for the approximation of dynamic viscosity

$$\mu = \rho \left[\frac{\alpha \rho_g}{\mu_g} + \frac{(1-\alpha)\rho_l}{\mu_l} \right]^{-1}. \tag{16}$$

TABLE 1

Experimental database. (a) steam-water tests; (b) air-water tests; (c) Freon tests

Reference	α	D (m)	L (m)	φ	a_n	ρ_l (kg/m^3)	ρ_g (kg/m^3)	M_n (kg)	V (m/s)	f_n (Hz)	ζ_n	y_n (μm)	Φ_F (N^2/Hz)
Axisa *et al.* (1984)	0·92	0·019	1·2	1	0·8	890	10	0·2	4	75	0·01	7	–
	0·92	0·019	1·2	1	0·8	890	10	0·2	7	75	0·01	20	–
	0·85	0·019	1·2	1	0·8	890	10	0·2	4	75	0·015	14	–
	0·85	0·019	1·2	1	0·8	890	10	0·2	8	75	0·015	35	–
	0·51	0·019	1·2	1	0·8	890	10	0·2	5	75	0·03	28	–
Nakamura *et al.* (1992)	0·90	0·019	0·17	–	1·1	915	2·5	–	11	2	–	–	5×10^{-2}
	0·90	0·019	0·17	–	1·1	915	2·5	–	11	5	–	–	3×10^{-2}
	0·90	0·019	0·17	–	1·1	915	2·5	–	11	30	–	–	2×10^{-3}
	0·90	0·019	0·17	–	1·1	825	15	–	11	2	–	–	1×10^{-2}
	0·90	0·019	0·17	–	1·1	825	15	–	11	5	–	–	4×10^{-3}
	0·90	0·019	0·17	–	1·1	825	15	–	11	30	–	–	1×10^{-4}
	0·90	0·019	0·17	–	1·1	760	30	–	11	2	–	–	4×10^{-3}
	0·90	0·019	0·17	–	1·1	760	30	–	11	5	–	–	1×10^{-3}
	0·90	0·019	0·17	–	1·1	760	30	–	11	30	–	–	3×10^{-5}
Hirota *et al.* (1996)	0·70	0·022	0·16	1	2	760	30	0·2	3	20	0·01	150	–
	0·90	0·022	0·16	1	2	825	15	0·2	2.5	20	0·01	100	–
	0·90	0·022	0·16	1	2	915	2.5	0·2	2	20	0·01	150	–

(a)

Reference	α	D (m)	L (m)	φ	a_n	ρ_l (kg/m^3)	ρ_g (kg/m^3)	M_n (kg)	V (m/s)	f_n (Hz)	ζ_n	y_n (μm)	Φ_F (N^2/Hz)
Axisa *et al.* (1989)	0·25	0·03	0·3	1	0·77	1000	1	0·14	1	20	0·09	70	–
	0·25	0·03	0·3	1	0·77	1000	1	0·14	2·7	20	0·025	350	–
	0·55	0·03	0·3	1	0·77	1000	1	0·09	4	25	0·1	260	–
	0·55	0·03	0·3	1	0·77	1000	1	0·09	9	30	0·02	1600	–
	0·95	0·03	0·3	1	0·77	1000	1	0·05	7	30	0·03	200	–
	0·95	0·03	0·3	1	0·77	1000	1	0·05	14	30	0·03	300	–
Taylor (1994)	0·15	0·013	0·6	1	0·5	1000	1	0·08	0·3	30	0·02	15	–
	0·15	0·013	0·6	1	0·5	1000	1	0·08	0·6	30	0·02	15	–
	0·50	0·013	0·6	1	0·5	1000	1	0·08	0·2	30	0·04	30	–
	0·50	0·013	0·6	1	0·5	1000	1	0·08	0·9	30	0·04	70	–
	0·94	0·013	0·6	1	0·5	1000	1	0·05	0·5	30	0·025	100	–
	0·94	0·013	0·6	1	0·5	1000	1	0·05	1·7	30	0·025	300	–

TABLE 1 (continued)

Experimental database. (a) steam-water tests; (b) air-water tests; (c) Freon tests

Reference	α	D (m)	L (m)	φ	a_n	ρ_l (kg/m^3)	ρ_g (kg/m^3)	M_n (kg)	V (m/s)	f_n (Hz)	ζ_n	y_n (μm)	Φ_F (N^2/Hz)
Nakamura *et al.*	0·77	0·019	0·012	–	0·12	1000	1	–	4	10	–	–	3×10^{-5}
(1992)	0·77	0·019	0·012	–	0·12	1000	1	–	4	100	–	–	3×10^{-7}
	0·93	0·019	0·012	–	0·12	1000	1	–	14	10	–	–	2×10^{-4}
	0·93	0·019	0·012	–	0·12	1000	1	–	14	100	–	–	2×10^{-6}

(b)

Reference	α	D (m)	L (m)	φ	a_n	ρ_l (kg/m^3)	ρ_g (kg/m^3)	M_n (kg)	V (m/s)	f_n (Hz)	ζ_n	y_n (μm)	Φ_F (N^2/Hz)
Feenstra *et al.*	0·50	0·006	0·3	1	0·5	1450	9	0·015	0·2	35	0·03	50	–
(1996)	0·90	0·006	0·3	1	0·5	1450	9	0·015	1	35	0·03	70	–
	0·40	0·006	0·3	1	0·5	1450	9	0·015	0·4	35	0·03	80	–
	0·85	0·006	0·3	1	0·5	1450	9	0·015	1·5	35	0·03	110	–
Pettigrew &	0·40	0·013	0·6	1	0·9	1200	42	0·1	0·3	25	0·03	40	–
Taylor (1994b)	0·40	0·013	0·6	1	0·9	1200	42	0·1	0·6	25	0·03	60	–
	0·80	0·013	0·6	1	0·9	1200	42	0·1	0·4	25	0·04	50	–
Axisa & Villard	0·85	0·013	1·2	0·84	0·11	1270	65	0·24	0·23	20	0·05	20	–
(1992)	0·85	0·013	1·2	0·84	0·11	1270	65	0·24	1	20	0·01	70	–
	0·75	0·013	1·2	0·84	0·11	1270	65	0·24	1·3	20	0·005	40	–
	0·95	0·013	1·2	0·84	0·11	1270	65	0·24	1	20	0·02	10	–
	0·85	0·013	0·57	0·9	0·08	1270	65	0·34	0·26	48	0·05	0·5	–
	0·85	0·013	0·57	0·9	0·08	1270	65	0·34	1·7	48	0·04	4	–
	0·75	0·013	0·57	0·9	0·08	1270	65	0·34	1	48	0·05	2	–
	0·95	0·013	0·57	0·9	0·08	1270	65	0·34	0·8	48	0·02	1	–
Gay *et al.*	0·58	0·01	0·144	1	1·5	1000	50	0·17	3·3	35	0·015	20	–
(1988)	0·84	0·01	0·144	1	1·5	1000	50	0·17	3·3	35	0·015	20	–
	0·84	0·01	0·144	1	1·5	1000	50	0·17	6·6	35	0·008	60	–
Delenne *et al.*	0·10	0·022	0·25	1	2	1000	50	0·45	2	30	0·03	70	–
(1997)	0·50	0·022	0·25	1	2	1000	50	0·37	3	35	0·03	50	–
	0·50	0·022	0·25	1	2	1000	50	0·37	4	35	0·01	200	–
	0·92	0·022	0·25	1	2	1000	50	0·3	3	35	0·01	30	–
	0·92	0·022	0·25	1	2	1000	50	0·3	6	35	0·005	200	–

(c)

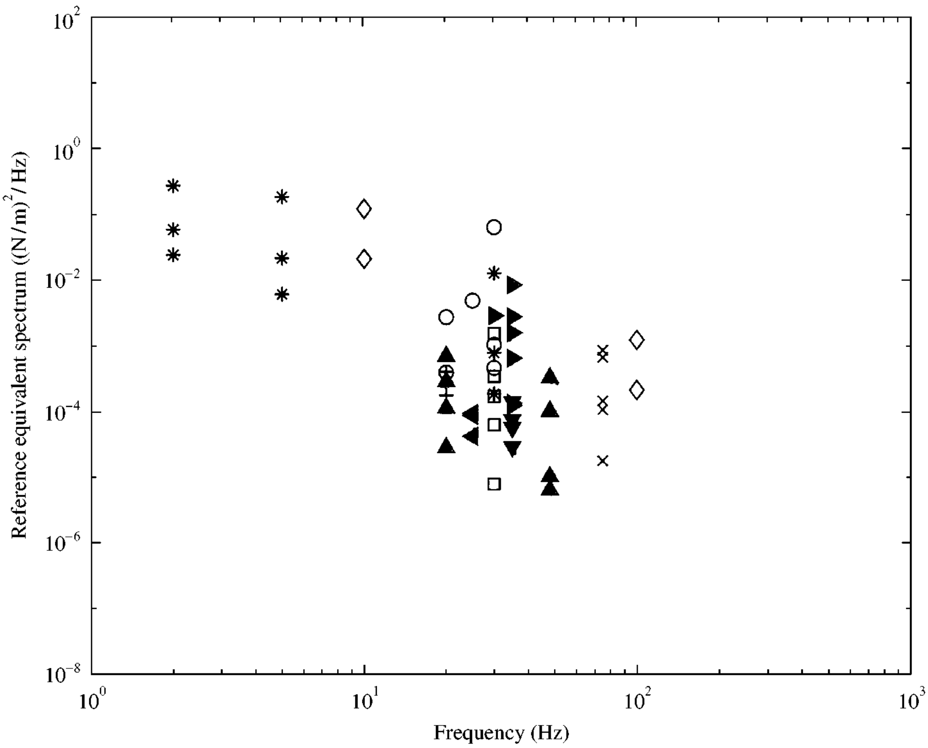

Figure 1. Reference equivalent spectrum. Steam-water tests; × Axisa *et al.* (1984); ∗ Nakamura *et al.* (1992); +, Hirota *et al.* (1996). Air-water tests: ○, Axisa *et al.* (1989); □, Taylor (1994); ◇, Nakamura *et al.* (1992). Freon tests: ▼, Feenstra *et al.* (1996); ◀, Pettigrew & Taylor (1994b); ▲, Axisa & Villard (1992); ▶ Gay *et al.* (1988), Delenne *et al.* (1997).

This allows the definition of a scaling factor $p_0 = \mu V/D$. It is observed in Figure 3 that the collapse of the data is not significantly improved.

Similarly, surface tension σ is used to scale the data through $p_0 = \sigma/D$. The result is shown in Figure 4. Due to significant differences in surface tensions, the data corresponding to the various mixtures are still widely separated. More precisely, all tests made with Freon R-22 and R-114, which have lower surface tensions are shifted above the others. Respective values of μ_l, μ_g and σ are listed in Table 2 for the various mixtures.

A more systematic search of the proper scaling parameters was undertaken, part of which is reported in de Langre *et al.* (1995), on data available at that time. Out of the known physical parameters, more than 20 combinations were used to build dimensionally acceptable expressions for f_0 and p_0, respectively. The scaling

$$f_0 = V/D_w, \tag{17}$$

$$p_0 = \rho_l g D_w, \tag{18}$$

with

$$D_w = 0{\cdot}1D/\sqrt{1-\alpha}, \tag{19}$$

shown in Figure 5 appears to result in a reasonable collapse of the data, and will be used further for the definition of an upper bound for the buffeting forces.

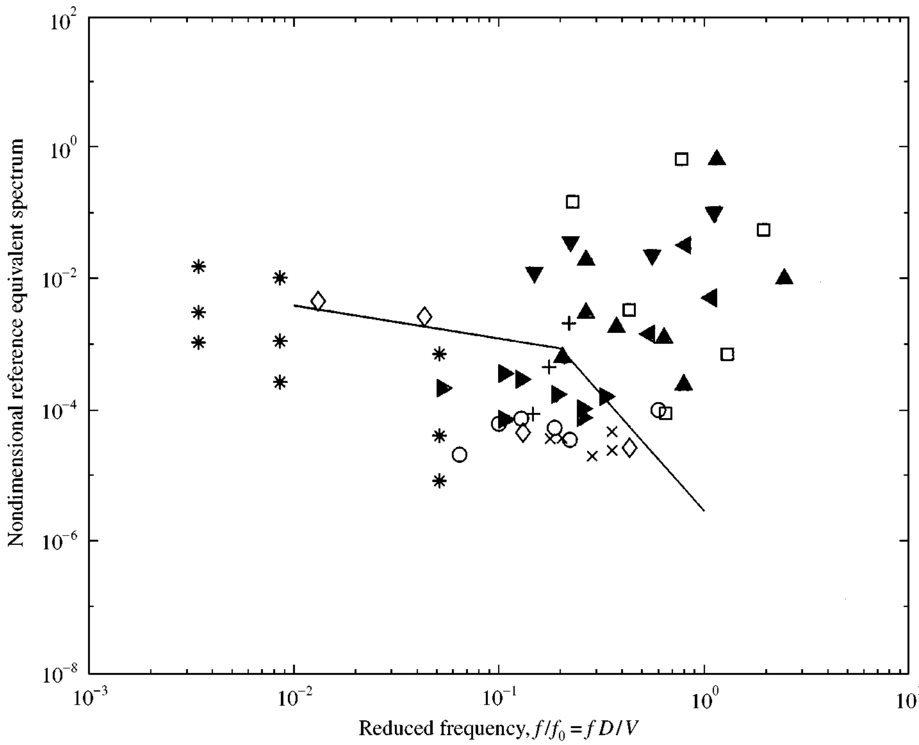

Figure 2. Nondimensional reference equivalent spectrum using single-phase scaling factors $f_0 = V/D$ and $p_0 = \frac{1}{2}\rho V^2$. Steam–water tests: ×, Axisa *et al.* (1984); ∗ Nakamura *et al.* (1992); +, Hirota *et al.* (1996). Air-water tests: ○, Axisa *et al.* (1984); □, Taylor (1994); ◇, Nakamura *et al.* (1992). Freon tests: ▼, Feenstra *et al.* (1996); ◀, Pettigrew & Taylor (1994b); ▲, Axisa & Villard (1992); ▶, Gay *et al.* (1988), Delenne *et al.* (1997). —, Upper bound for single-phase-equivalent spectrum (Axisa *et al.* 1990).

5. DISCUSSION

5.1. Physical Meaning of the Scaling Parameters

In the frequency scale of equation (17), $f_0 = V/D_w$, the choice of the velocity V is quite straightforward and consistent with previous work in single-phase configurations. It should be mentioned that in the experimental data considered here, the reduced frequency $f_R = f/f_0$ is varied either through changes of the flow velocity at a given frequency, or through changes of the frequency at a prescribed flow velocity. This is made possible by the use of data sets obtained from both direct and indirect measurements of the forces.

The use of a length scale D_w, equation (19), which is not a dimension of the vibrating system, is specific to two-phase flows. It was introduced by Taylor (1992) as inherent to the flow structure and its natural inhomogeneity. To introduce such a characteristic void length related to the two-phase mixture, an empirical correlation was originally proposed by Taylor (1992, 1996) as:

$$D_b = 0{\cdot}00163\sqrt{V}/\sqrt{1-\alpha}. \tag{20}$$

If one excludes the small dependence on the flow velocity and includes the tube diameter as a length scale, with a proportionality factor to adjust the magnitude, equation (20) reduces

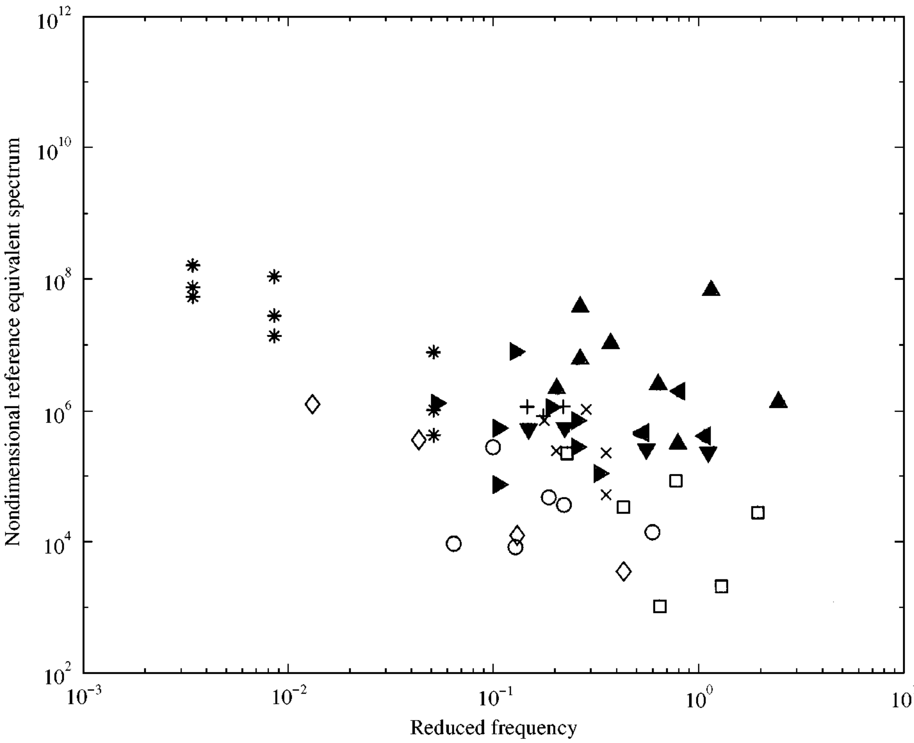

Figure 3. Nondimensional reference equivalent spectrum using viscosity as scaling parameter $p_0 = \mu V/D$ and $f_0 = V/D$. Steam–water tests: ×, Axisa *et al.* (1984); ∗ Nakamura *et al.* (1992); +, Hirota *et al.* (1996). Air-water tests: ○, Axisa *et al.* (1989); □, Taylor (1994); ◇, Nakamura *et al.* (1992). Freon tests: ▼, Feenstra *et al.* (1996); ◀, Pettigrew & Taylor (1994b); ▲, Axisa & Villard (1992); ▶ Gay *et al.* (1988), Delenne *et al.* (1997).

to the simplified form of equation (19). This empirical void length will be referred to as a bubble size, though there might not be any bubbles present but other gas–liquid patterns. This is indeed a very rough approximation of the complex statistical distribution of the liquid and gaseous phases around the tubes. Much work remains to be done on the relationship between the actual mixture surrounding the tube and the excitation mechanism (Lian *et al.* 1997; Joo & Dihr 1994). For instance, such a scale of bubble size should be affected by surface tension, flow regimes, pressure, and even geometry of the array. The reduction of the data might be improved by taking these effects into consideration. Here, equation (19) was found to be of sufficient accuracy for predictive analysis.

The force scaling parameter, $p_0 = \rho_l g D_w$, only chosen here because of its efficiency in collapsing the data, may give some insight as to the underlying excitation mechanism. Gravity has a well-known influence on the structure of two-phase flows, as it induces strong buoyancy forces on the gaseous phase (Delhaye *et al.* 1981). Typically, in drift models, buoyancy forces are counterbalanced by friction between the phases, thus implicitly defining a difference between the velocities of gas and liquid. Gravity in the scaling factor might therefore be related to a dynamic pressure built out of such drift velocities. In other words, buffeting forces in two-phase flow seem to depend on the difference between the velocities of each phase and not on the average velocity of the mixture.

Other researchers (Lian *et al.* 1997; Papp & Chen 1994; Nakamura *et al.* 1992; Goyder 1988) have considered scaling their data with the help of various kinds of dynamic pressures.

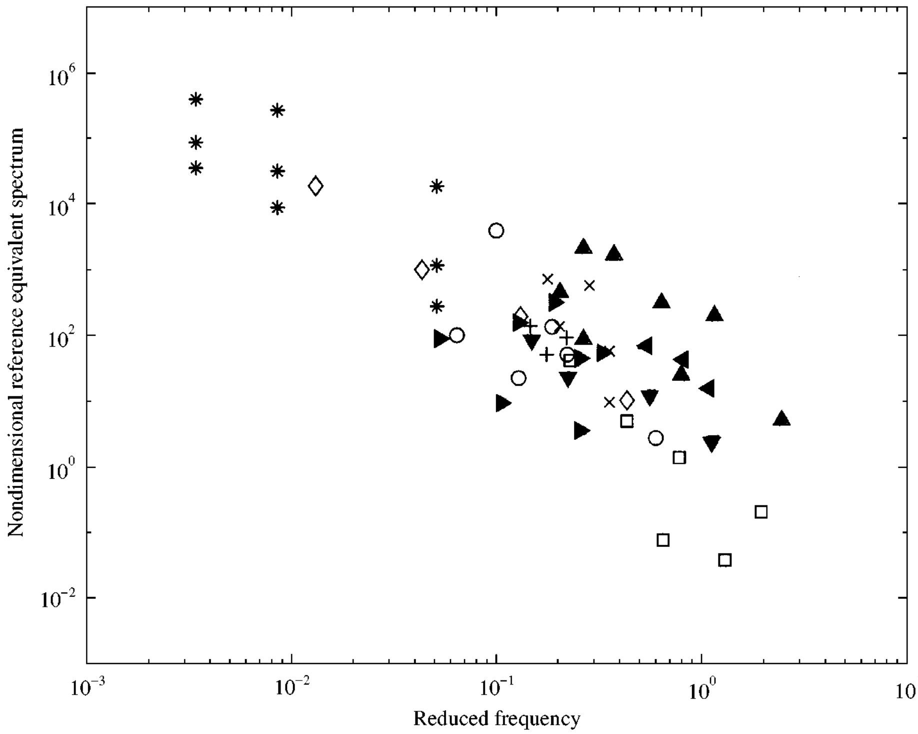

Figure 4. Nondimensional reference equivalent spectrum using surface tension as scaling parameter $p_0 = \sigma/D$ and $f_0 = V/D$. Steam–water tests: ×, Axisa *et al.* (1984); ∗ Nakamura *et al.* (1992); +, Hirota *et al.* (1996). Air-water tests: ○, Axisa *et al.* (1989); □, Taylor (1994); ◇, Nakamura *et al.* (1992). Freon tests: ▼, Feenstra *et al.* (1996); ◀, Pettigrew & Taylor (1994b); ▲, Axisa & Villard (1992); ▶ Gay *et al.* (1988), Delenne *et al.* (1997).

Table 2

Physical parameters relative to the various two-phase mixtures

Fluid mixture	μ_l (Pa s)	μ_g (Pa s)	σ (N/m)
Air/water	1×10^{-3}	2×10^{-5}	7×10^{-2}
Steam/water	1×10^{-4}	2×10^{-5}	2×10^{-2}
R–11	4×10^{-4}	1×10^{-5}	2×10^{-2}
R–22	2×10^{-4}	2×10^{-5}	8×10^{-3}
R–114	2×10^{-4}	1×10^{-5}	5×10^{-3}
R–13B1/water	1×10^{-3}	2×10^{-5}	7×10^{-2}

These scalings remain largely open to discussion. The scope of the present paper is not to present an alternative model of the physical excitation mechanism. However, the proposed scaling factors may provide hints for further research directions.

5.2. Effect of Flow Regime

It has been often pointed out that two-phase flow across the bundles does not occur in the same flow regime within the range of flow parameters of practical interest. Yet, accurate

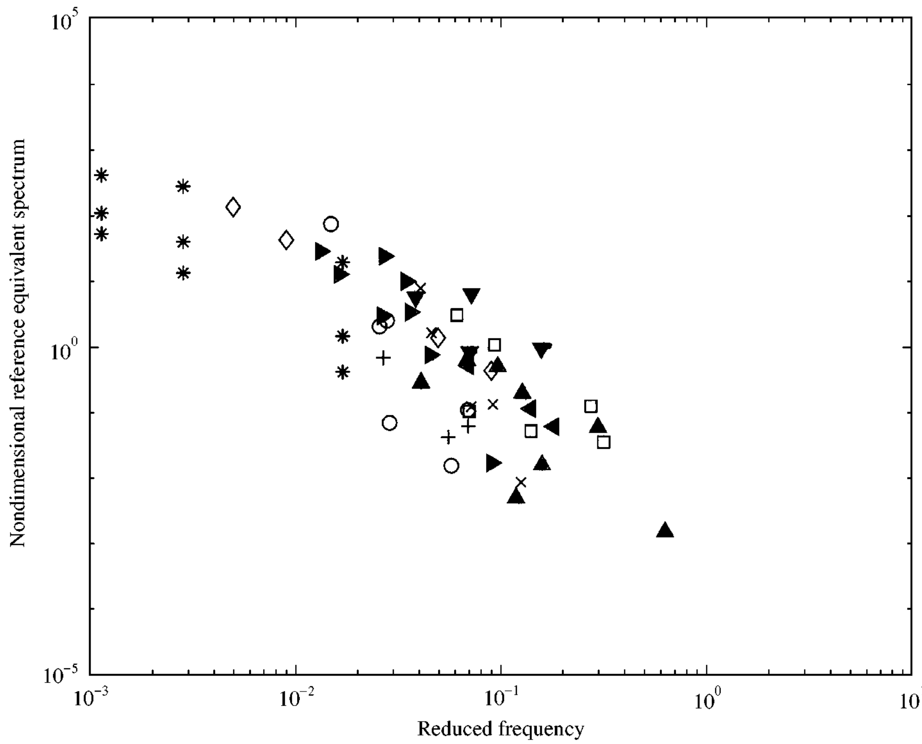

Figure 5. Nondimensional reference equivalent spectrum using gravity as scaling parameter $p_0 = \rho_l g D_w$ and $f_0 = V/D_w$. Steam–water tests: ×, Axisa *et al.* (1984); ∗ Nakamura *et al.* (1992); +, Hirota *et al.* (1996). Air-water tests: ○, Axisa *et al.* (1989); □, Taylor (1994); ◇, Nakamura *et al.* (1992). Freon tests: ▼, Feenstra *et al.* (1996); ◀, Pettigrew & Taylor (1994b); ▲, Axisa & Villard (1992); ▶ Gay *et al.* (1988), Delenne *et al.* (1997).

transition criteria between the different kinds of regimes are not available. Flow-pattern maps may nevertheless help to understand differences between cases. In recent papers (Feenstra *et al.* 1995; Taylor *et al.* 1996), it has been shown that some qualitative changes in terms of flow-induced vibrations could be related to crossing of boundaries in flow-pattern maps. This is a promising development, but the accurate knowledge of the flow structure in cross-flow still requires considerable experimental effort.

As of today, it is not unanimously accepted that any of the commonly used maps (Grant, Taitel, Mc Quillan & Whalley) gives an accurate description of the flow régimes across a tube bundle for the various fluids considered here. In spite of these limitations, we show in Figure 6 some physical parameters which are agreed to be relevant in terms of flow régimes, namely void fraction α and velocity V, as well as the superficial velocities of the phases $V_{Sl} = (1-\alpha)V$ and $V_{Sg} = \alpha V$. Though the boundaries between régimes are not plotted, it can reasonably be assumed that the range of thermohydraulic parameters of the database most probably covers several kinds of flow regimes. Though it is also most likely that crossing a flow regime boundary, wherever it actually stands, does affect buffeting excitation, it has not been found necessary here to separate the data into several groups as part of the reduction procedure. Moreover, for practical applications in predictive analysis, the knowledge of the local flow regime in a given heat exchanger is not yet accessible with an acceptable level of confidence. So, as a first approximation, we do not consider that flow regimes play a major role in the random excitation. This may not be true if one considers a more refined scale on a smaller given set of experiments [see Taylor (1996)]. Furthermore,

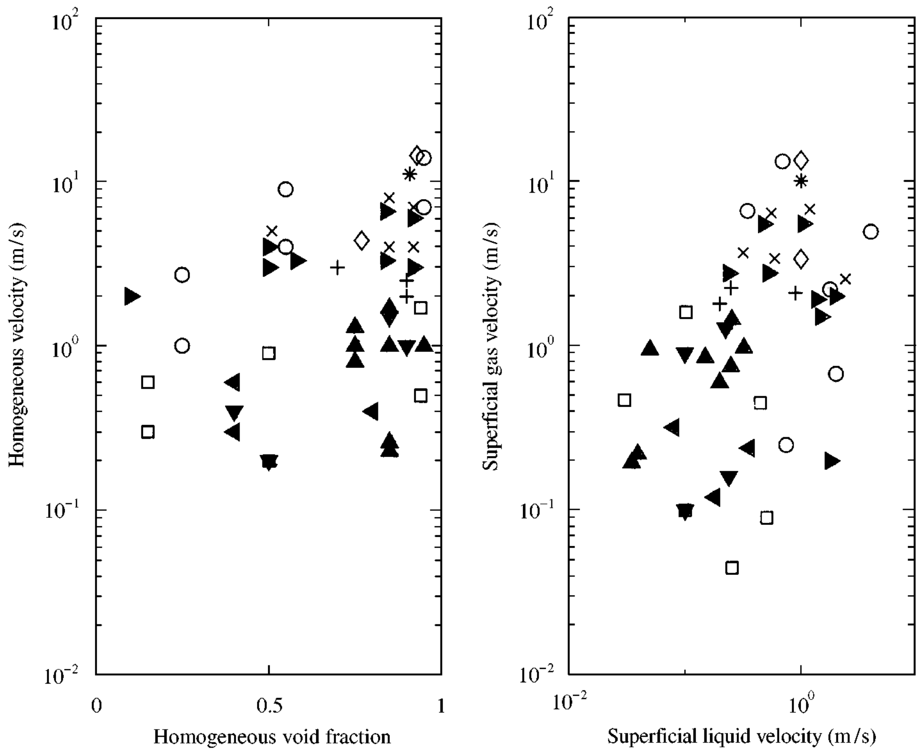

Figure 6. Thermohydraulics parameters of the experimental data. Steam–water tests: ×, Axisa *et al.* (1984); ∗ Nakamura *et al.* (1992); +, Hirota *et al.* (1996). Air-water tests: ○, Axisa *et al.* (1989); □, Taylor (1994); ◇, Nakamura *et al.* (1992). ○ Freon tests: ▼, Feenstra *et al.* (1996); ◀, Pettigrew & Taylor (1994b); ▲, Axisa & Villard (1992); ▶ Gay *et al.* (1988), Delenne *et al.* (1997).

this may not be applicable at all to other phenomena of flow-induced vibrations in two-phase flow, such as fluid-elastic forces or damping.

5.3. Effect of Fluid Mixture

When a given tube bundles is tested with two different mixtures (Axisa *et al.* 1990; Pettigrew & Taylor 1994a; Hirota *et al.* 1997), the various fluid mixtures seem to lead to buffeting forces of the same order of magnitude. This also appears here in the fact that the scaling parameters that are retained do not strongly differentiate between mixtures. In Figure 7, no clear difference appears in the spectra of the different mixtures, except a slightly lower level for the high pressure steam-water tests. Hence, in the following section, it is not found necessary to explicitly differentiate between the fluids. Again this may not be applicable to other phenomena in two-phase flow-induced vibrations.

6. PROPOSED UPPER BOUND

The preceding results on the gathering of experimental data when proper scaling is used lead us to propose an upper bound of the dimensionless reference equivalent spectrum of random forces $[\overline{\Phi_E^0}]_U$, defined as follows:

$$\Phi_E^0(f) = \left[(\rho_l g D_w D)^2 \frac{D_w}{V} \right] [\overline{\Phi_E^0}]_U, \tag{21}$$

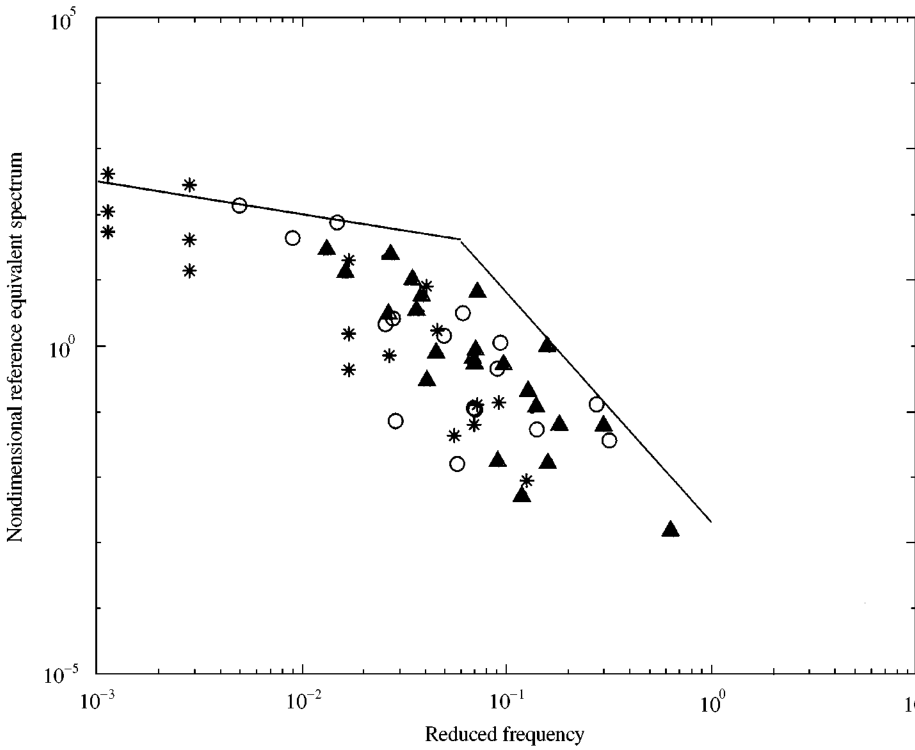

Figure 7. Proposed upper bound (——); * steam-water tests; ○ air-water tests; ▲ Freon tests.

where $D_w = 0{\cdot}1D/\sqrt{1-\alpha}$; the reference lengths are $L_0 = 1$ m, $D_0 = 0{\cdot}02$ m, and

$$[\overline{\Phi_E^0}]_U = 10 f_R^{-0{\cdot}5} \qquad \text{if} \quad 10^{-3} \le f_R \le 0{\cdot}06, \tag{22}$$

$$[\overline{\Phi_E^0}]_U = 2{\cdot}10^{-3} f_R^{-3{\cdot}5} \qquad \text{if} \quad 0{\cdot}06 \le f_R \le 1, \tag{23}$$

using as reduced frequency $f_R = fD_w/V$. The corresponding curve is shown in Figure 7. This upper bound differs from previous ones in its nondimensional form and, of more importance, in the larger sets of experimental data that support it.

If one considers the data sets used to derive it, the present upper bound seems applicable for homogeneous void fractions from 10 to 95%, homogeneous velocities from 0·2 to 14 m/s, bubbly or churn flow and most commonly used liquid-gas mixtures. For very high void fractions (more than 95%), one should consider that the value of the void fraction becomes much more dependent on the flow model used and that the physical mechanism might be different. For very low void fractions (less than 10%), few experimental data are accessible, and mostly on single tubes (Hara 1987). A model that would link these extreme ranges of void fractions to single-phase liquid or gaseous situations still remains to be done.

The practical use of the upper bound defined in equations (22) and (23) requires the same procedures as in the single-phase case (de Langre *et al.* 1991). If the mechanical boundary conditions of the tube are linear, the r.m.s. displacement contributed by mode n is readily obtained by combining equations (12) and (21). One finds

$$y_n(s) = \left[\frac{\varphi_n^2(s) L^2 a_n}{64\pi^3 f_n^3 M_n^2 \zeta_n} (\rho_l g D_w D)^2 \frac{D_w}{V} \left(\frac{L_0}{L}\right) \left(\frac{D}{D_0}\right) [\overline{\Phi_E^0}]_U \right]^{1/2}. \tag{24}$$

As the flow velocity is increased, y_n varies as $V^{1\cdot25}$ or $V^{-0\cdot25}$ depending on the velocity range. This very slow growth, as compared to the single-phase situation, is indeed related to the absence of the velocity in the pressure scaling factor. Typical values for a steam generator U-tube ($f = 30$ Hz, $D = 0{\cdot}02$, $\alpha = 0{\cdot}8$) yield that y_n would have a weak dependence on velocity above 2.2 m/s. It would even decrease if a compensating decrease in damping did not usually occur as a result of fluid-elastic forces. This is in agreement with several observations (Axisa *et al.* 1984; Taylor *et al.* 1995). If the boundary conditions include impacting and friction on supports, the solution of the dynamic equations of motion requires a time-stepping integration. Specific procedures for the generation of adequate random forces are then to be used (Axisa *et al.* 1988; de Langre *et al.* 1991).

7. CONCLUSION

A large set of experimental measurements of buffeting forces in two-phase flows has been considered. This data base is made of results from eleven different experimental programs in five different laboratories over the past 15 years. Straight or U-tube bundles, flexible or rigid, have been included, under cross-flow of steam-water, air–water or various Freon mixtures, the void fractions of which range from 10 to 95%. The primary intent of this work was to define an upper bound on the magnitude of these buffeting forces for practical applications in predictive analysis of flow-induced vibrations in heat-exchangers. This required a preliminary investigation of the appropriate dimensionless expression for the buffeting force spectrum. The principal conclusions drawn from this study are the following.

(a) The various measurements may be analysed with a formalism similar in principle to that successfully used for single-phase buffeting. It clearly appears that these buffeting forces may not be scaled in the same manner as in single-phase flows. In other words, the dynamic pressure does not control the magnitude of these forces, which confirms previous results by other authors.

(b) When other scaling parameters are explored, it is found that neither viscosity nor surface tension appear as acceptable to define a dimensionless spectrum.

(c) Scaling factors based on gravity forces and a void length provide a reasonable collapse of the data. Though these parameters are not chosen through a physical modelling of the interaction between the flow and the tube, it is believed that they may lead to a better understanding of this phenomenon.

(d) At that first level of approximation, the particular flow regime or nature of the liquid and gaseous phases do not play a major role over the entire database.

(e) An upper bound on the magnitude of these buffeting forces is proposed, with a high level of confidence as a result of the large database considered here.

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APPENDIX: NOMENCLATURE

A	free-stream cross-sectional area (m^2)
a_n	modal correlation factor
D	tube diameter (m)
D_0	tube reference diameter (m)
D_w	empirical void length determined by bubble size
D_b	characteristic void length
F	force per unit length (N/m)
f_0	frequency scaling factor (Hz)
f_n	modal frequency (Hz)
L	tube length (m)
L_0	tube reference length (m)
M_n	modal mass (kg)

p_0	pressure scaling factor (Pa)
P	pitch (m)
Q	volume flow rate (m^3/s)
Q_g	gas-phase volume flow rate (m^3/s)
Q_l	liquid-phase volume flow rate (m^3/s)
s	space variable (m)
V	homogeneous gap flow velocity (m/s)
V_{Sl}	superficial liquid velocity (m/s)
V_{Sg}	superficial gas velocity (m/s)
y_n	r.m.s. modal displacement (m)
α	homogeneous void fraction
ζ_n	modal damping
λ_c	length of correlation (m)
μ	mixture dynamic viscosity (Pas)
μ_g	gas dynamic viscosity (Pas)
μ_l	liquid dynamic viscosity (Pas)
ρ_g	density of gaseous phase (kg/m^3)
ρ_l	density of liquid phase (kg/m^3)
ρ	density of mixture (kg/m^3)
σ	surface tension (N/m)
φ_n	modal shape
Φ	autoco rrelation spectrum ($(N/m)^2/Hz$)
Φ_E	equivalent spectrum ($(N/m)^2/Hz$)
Φ_E^0	reference equivalent spectrum ($(N/m)^2/Hz$)
$\overline{\Phi_E^0}$	dimensionless reference equivalent spectrum
$[\overline{\Phi_E^0}]_U$	upper bound of dimensionless reference equivalent spectrum
Ψ_F	cross-correlation spectrum of forces per unit length ($(N/m)^2/Hz$)